PERFECT RECONSTRUCTION QMF STRUCTURES WHICH YIELD LINEAR PHASE FIR ANALYSIS FILTERS

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Abstract: We present a perfect reconstruction FIR linear phase *lattice* structure for the two channel QMF bank. Furthermore, this structure covers most pairs of perfect reconstruction FIR linear phase analysis filters which have the same (odd) order. For general M, we derive conditions which any QMF perfect reconstruction linear phase structure must obey. A design example is presented for the $M = 2$ case.

I. Introduction

Quadrature mirror filters (in short QMF) are used in many speech and communication applications [1],[2]. The theory of perfect reconstruction when M is a power of two is well known [2],[3], and some methods for perfect reconstruction for arbitrary M have been recently reported [4-6]. The FIR analysis and synthesis filters in the above structures are unrestricted in the sense that they do not possess any special propeties besides the perfect reconstruction one. In a recent paper [5], a procedure for designing two-channel perfect reconstruction systems with linear-phase FIR filters has been outlined. This procedure is based on judicious factorization of a linear-phase FIR halfband filter. The number of possible spectral factors grows exponentially with respect to the order of the filters and furthermore, the resulting filters are not guaranteed to be optimal.

Let $H_k(z)$, $0 \le k \le M-1$ and $F_k(z)$, $0 \le k \le M-1$ denote, respectively, the analysis and synthesis filters of a M-channel maximally decimated QMF bank. Following the procedure in [4], we represent $H_k(z)$ and $F_k(z)$ in terms of their polyphase components as $H_k(z) = \sum_{\ell=0}^{M-1} z^{-\ell} E_{k,\ell}(z^M)$ and $F_k(z) = \sum_{\ell=0}^{M-1} z^{-(M-1-\ell)} R_{\ell,k}(z^M)$. With $\mathbf{E}(z) \triangleq [E_{k,\ell}(z)]$ and $\mathbf{R}(z) \triangleq [R_{\ell,k}(z)]$ we can represent the QMF bank as in Fig. 10a of [4]. Assuming that $H_k(z)$ are FIR, we can obtain a perfect reconstruction system with FIR synthesis filter, if $\det \mathbf{E}(z) = cz^{-r}$ where $c \neq 0$ is a constant and r is a positive integer. In this paper the term 'perfect reconstruction' is synonymous to $\det \mathbf{E}(z) = cz^{-r}$.

We shall assume that all the analysis filters have the same order, and denote it by $N-1$. We shall also assume for simplicity, that N is a multiple of M. We begin with general M and show that the FIR linear-phase analysis filters of any perfect reconstruction FIR QMF structure must satisfy the following properties:

- For even M, there are $M/2$ symmetric and $M/2$ anti-symmetric linear-phase analysis filters.
- For odd M, the number of symmetric linear-phase filters is necessarily one more than the number of anti-symmetric linear-phase filters. Moreover, $N-1$ has to be even.

We then present a structure with first order building blocks, using which we can impose the perfect reconstruction condition. For the special case of two channels, we present a perfect reconstruction QMF *lattice* structure which covers most pairs of linear-phase perfect reconstruction analysis filters that have the same odd order. We also show that most pairs of linear phase perfect reconstruction analysis filters can be synthesized using the proposed lattice structure.

Notations: Bold faced letters denote both vectors and matrices in this paper. $\hat{\mathbf{H}}(z) = z^{-(N-1)}\mathbf{H}(z^{-1})$ where $N-1$ is the order of $\mathbf{H}(z)$. This convention also holds for scalar matrices. We abbreviate linear-phase as LP and perfect reconstruction as PR.

II. Properties of PR systems which yield LP analysis filters.

Without loss of generality, suppose that the first K analysis filter impulse responses are symmetric and the remaining $M-K$ are anti-symmetric. In other words,

$$H_\ell(z) = \begin{cases} \hat{H}_\ell(z), & 0 \le \ell \le K-1; \\ -\hat{H}_\ell(z), & K \le \ell \le M-1. \end{cases} \tag{1}$$

Using polyphase filters notation, (1) becomes

$$\mathbf{E}(z^M)\begin{pmatrix} 1 \\ \vdots \\ z^{-(M-1)} \end{pmatrix} = \mathbf{J}z^{-(N-1)}\mathbf{E}(z^{-M})\begin{pmatrix} 1 \\ \vdots \\ z^{(M-1)} \end{pmatrix}$$

$$= \mathbf{J}z^{-(N-M)}\mathbf{E}(z^{-M})\begin{pmatrix} z^{-(M-1)} \\ \vdots \\ 1 \end{pmatrix},$$

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where $\mathbf{J} = \begin{pmatrix} \mathbf{I}_K & \mathbf{0} \\ \mathbf{0} & -\mathbf{I}_{M-K} \end{pmatrix}$. Comparing like powers of both sides, we conclude

$$E_{\ell,i}(z) = \begin{cases} \hat{E}_{\ell,M-1-i}(z), & 0 \leq \ell \leq K-1; \\ -\hat{E}_{\ell,M-1-i}(z), & K \leq \ell \leq M-1. \end{cases} \quad (2)$$

$\mathbf{E}(z)$ satisfying (2) can be written as

$$\mathbf{E}(z) = \begin{cases} \mathbf{E}'(z)\begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P}_0 \end{pmatrix}, & \text{odd } M; \\ \mathbf{E}'(z)\begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}_0 \end{pmatrix}, & \text{even } M, \end{cases} \quad (3)$$

where

$$\mathbf{E}'(z) = \begin{cases} \begin{pmatrix} \mathbf{Q}_0(z) & \mathbf{Q}_1(z) & \hat{\mathbf{Q}}_0(z) \\ \mathbf{Q}_2(z) & \mathbf{Q}_3(z) & -\hat{\mathbf{Q}}_2(z) \end{pmatrix}, & \text{odd } M; \\ \begin{pmatrix} \mathbf{Q}_0(z) & \hat{\mathbf{Q}}_0(z) \\ \mathbf{Q}_2(z) & -\hat{\mathbf{Q}}_2(z) \end{pmatrix}, & \text{even } M. \end{cases} \quad (4)$$

The dimensions of $\mathbf{Q}_0(z)$, $\mathbf{Q}_1(z)$, $\mathbf{Q}_2(z)$ and $\mathbf{Q}_3(z)$ are $(K \times L)$, $(K \times 1)$, $(M-K) \times L$ and $(M-K) \times 1$ respectively. $\mathbf{P}_0$ in (3) is defined to be $\begin{pmatrix} 0 & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & 0 \end{pmatrix}_{L \times L}$, and $L = \begin{cases} M/2, & M \text{ even}; \\ (M-1)/2, & M \text{ odd}. \end{cases}$ As observed from (2), $\mathbf{Q}_1(z)$ and $\mathbf{Q}_3(z)$ are symmetric and anti-symmetric LP polynomial matrices respectively. Having found the necessary form of $\mathbf{E}(z)$ for LP analysis filters, we investigate the additional conditions that $\mathbf{E}(z)$ should possess, so that the QMF bank is a PR structure. Let us be reminded that a FIR PR QMF structure has $\det \mathbf{E}(z) \neq 0$ for all z. Since $\mathbf{E}(z)$ is a cascade of $\mathbf{E}'(z)$ with a nonsingular constant matrix, to simplify the analysis we deal directly with $\mathbf{E}'(z)$ instead of $\mathbf{E}(z)$.

• Even M:

$$\det \mathbf{E}'(1) = \det \begin{array}{c} \\ K \\ M-K \end{array} \begin{array}{c} \quad L \qquad\quad L \\ \begin{pmatrix} \mathbf{Q}_0(1) & 2\mathbf{Q}_0(1) \\ \mathbf{Q}_2(1) & \mathbf{0} \end{pmatrix} \end{array}.$$

From the above equation $\det \mathbf{E}'(1) = 0$ if $K < L$. (5)

$$\text{Similarly,} \det \mathbf{E}'(1) = \det \begin{array}{c} \\ K \\ M-K \end{array} \begin{array}{c} \quad L \qquad\quad L \\ \begin{pmatrix} \mathbf{Q}_0(1) & \mathbf{0} \\ \mathbf{Q}_2(1) & 2\mathbf{Q}_2(1) \end{pmatrix} \end{array}.$$

Therefore, $\det \mathbf{E}'(1) = 0$ if $M-K < L$. (6)

From both (5) and (6), the LP PR FIR QMF structure must have $K = L = M/2$ for even M which implies that the number of symmetric and anti-symmetric LP analysis filters are the same.

• Odd M:

• Odd $N-M$: Since $\mathbf{Q}_3(z)$ is an anti-symmetric polynomial, it has a zero at $z=1$. Consequently, (4) yields

$$\det \mathbf{E}'(1) = \det \begin{array}{c} \\ K \\ M-K \end{array} \begin{array}{c} \quad L \qquad\quad 1 \qquad\quad L \\ \begin{pmatrix} \mathbf{Q}_0(1) & \mathbf{Q}_1(1) & 2\mathbf{Q}_0(1) \\ \mathbf{Q}_2(1) & \mathbf{0} & \mathbf{0} \end{pmatrix} \end{array}.$$

Thus, $\det \mathbf{E}'(1) = 0$ if $K < L+1$. (7)

Likewise, $\mathbf{Q}_1(z)$ has a zero at $z=-1$ since it is an odd order symmetric polynomial. Hence, at $z=-1$

$$\det \mathbf{E}'(-1) = \det \begin{array}{c} \\ K \\ M-K \end{array} \begin{array}{c} \quad L \qquad\quad 1 \qquad\quad L \\ \begin{pmatrix} \mathbf{Q}_0(-1) & \mathbf{0} & \mathbf{0} \\ \mathbf{Q}_2(-1) & \mathbf{Q}_3(-1) & 2\mathbf{Q}_2(-1) \end{pmatrix} \end{array}.$$

As a result, $\det \mathbf{E}'(-1) = 0$ if $M-K < L+1$. (8)

The two necessary conditions for nonsingularity of $\mathbf{E}'(z)$ obtained from (7) and (8) contradict each other so that LP PR structure is not possible here.

• Even $N-M$: The condition (7) still holds here since $\mathbf{Q}_3(1) = \mathbf{0}$. At $z=-1$,

$$\det \mathbf{E}'(-1) = \det \begin{array}{c} \\ K \\ M-K \end{array} \begin{array}{c} \quad L \qquad\quad 1 \qquad\quad L \\ \begin{pmatrix} \mathbf{Q}_0(-1) & \mathbf{Q}_1(-1) & \mathbf{0} \\ \mathbf{Q}_2(-1) & \mathbf{0} & 2\mathbf{Q}_2(-1) \end{pmatrix} \end{array}.$$

Thus, $\det \mathbf{E}'(-1) = 0$ if $M-K < L$. (9)

The only choice for K from (7) and (9) is $K = L+1$, or in other words, the number of the symmetric LP filters is one more than that of the anti-symmetric ones. In summary, the PR FIR structure that yields LP analysis filters satisfies $K = \begin{cases} L, & M \text{ even} \\ L+1, & M \text{ odd}. \end{cases}$ Furthermore, the order of $H_\ell(z)$ is even for odd M. $\mathbf{E}'(z)$ then takes the form as in (4) where the dimensions of $\mathbf{Q}_0(z)$,$\mathbf{Q}_1(z)$,$\mathbf{Q}_2(z)$ and $\mathbf{Q}_3(z)$ are $(M-L) \times L$, $(M-L) \times 1$, $(L \times L)$, and $(L \times 1)$ respectively. One could impose additional conditions on $\mathbf{E}'(z)$ in (4) to obtain additional properties for the structure. For instance, imposing lossless property [4] on $\mathbf{E}'(z)$ for even M yields a LP PR QMF structure. One possible LBR LP $\mathbf{E}'(z)$ has the form $\mathbf{E}'(z) = \begin{pmatrix} \mathbf{Q}_0(z) & \hat{\mathbf{Q}}_0(z) \\ \mathbf{Q}_0(z) & -\hat{\mathbf{Q}}_0(z) \end{pmatrix}$ where $\mathbf{Q}_0(z)$ is LBR.

The above approach of imposing additional conditions on $\mathbf{E}'(z)$ deals with the elements of $\mathbf{E}'(z)$ directly. On the other hand, we could decompose $\mathbf{E}'(z)$ in (4) into first order building blocks which makes it more convenient to impose additional conditions. Given $\mathbf{E}'_{S+1}(z)$ of order $S+1$, let us decompose it into a cascade of first order building blocks as follows:

$$\mathbf{E}'_{S+1}(z) = \mathbf{E}'_S(z)\mathbf{A}_S(z). \quad (10)$$

The strategy here is to find the first order block $\mathbf{A}_S(z)$ such that $\mathbf{E}'_S(z)$ has the same form as $\mathbf{E}'_{S+1}(z)$ where the order of $\mathbf{E}'_S(z)$ is S. The above factorization is not the most general one for arbitrary M. We elaborate only on the even M case here since the other case can be similarly derived.

Let the elements of $\mathbf{E}'_S(z)$ be $\mathbf{Q}_0^{(S)}(z)$ and $\mathbf{Q}_2^{(S)}(z)$. Substituting $\mathbf{E}'(z)$ in (4) into (10), we have

$$\mathbf{E}'_{S+1}(z) = \begin{pmatrix} \mathbf{Q}_0^{(S)}(z) & \hat{\mathbf{Q}}_0^{(S)}(z) \\ \mathbf{Q}_2^{(S)}(z) & -\hat{\mathbf{Q}}_2^{(S)}(z) \end{pmatrix} \mathbf{A}_S(z). \qquad (11)$$

We assume $\mathbf{E}'_S(z)$ to be of the form (4), and find the conditions to be imposed on $\mathbf{A}_S(z)$ so that $\mathbf{E}'_{S+1}(z)$ also has the form (4). From (11) we see that these conditions are

$$\mathbf{Q}_0^{(S)}(z)(\mathbf{A}_{01}(z) - \hat{\mathbf{A}}_{10}(z)) = \hat{\mathbf{Q}}_0^{(S)}(z)(\hat{\mathbf{A}}_{00}(z) - \mathbf{A}_{11}(z)),$$

$$\mathbf{Q}_2^{(S)}(z)(\mathbf{A}_{01}(z) - \hat{\mathbf{A}}_{10}(z)) = -\hat{\mathbf{Q}}_2^{(S)}(z)(\hat{\mathbf{A}}_{00}(z) - \mathbf{A}_{11}(z)).$$

Since we wish the above equations to hold for *any* $\mathbf{E}'_S(z)$ of the form (4), we constrain $\mathbf{A}_S(z)$ to be such that $\mathbf{A}_{01}(z) = \hat{\mathbf{A}}_{10}(z)$, and $\mathbf{A}_{11}(z) = \hat{\mathbf{A}}_{00}(z)$. $\mathbf{A}_S(z)$ then takes the form

$$\mathbf{A}_S(z) = \begin{pmatrix} \mathbf{A}_{00}(z) & \hat{\mathbf{A}}_{10}(z) \\ \mathbf{A}_{10}(z) & \hat{\mathbf{A}}_{00}(z) \end{pmatrix}. \qquad (12)$$

Continuing the process, we have $\mathbf{E}'_{S+1}(z) = \mathbf{E}'_1(z)\prod_{i=1}^{S} \mathbf{A}_i(z)$. The analysis bank in Fig. 10a of [4] thus becomes Fig. 1, where $\mathbf{A}_i(z)$ is as in (12) and

$$\mathbf{E}'_1(z) \triangleq \begin{pmatrix} \mathbf{Q}_0^{(1)}(z) & \hat{\mathbf{Q}}_0^{(1)}(z) \\ \mathbf{Q}_2^{(1)}(z) & -\hat{\mathbf{Q}}_2^{(1)}(z) \end{pmatrix} \qquad (13)$$

is a first order system. We have presented a structure which yields LP analysis filters for general M. We, however, have not imposed any additional conditions such as PR condition on the structure of Fig. 1. For the case of two channels, we will impose the PR condition on these first order building blocks to obtain a PR LP QMF *lattice* structure. Moreover, we also outline a synthesis procedure to show that most pairs of PR LP analysis filters which have the same odd order can be implemented using this lattice structure.

III. PR lattice structure for two channel LP analysis filter bank.

Since $L = M/2 = 1$ for this case, all matrices in (12) and (13) becomes scalars, i.e.,

$$\mathbf{A}_i(z) = \begin{pmatrix} A_{00}(z) & \hat{A}_{10}(z) \\ A_{10}(z) & \hat{A}_{00}(z) \end{pmatrix}, \mathbf{E}'_1(z) = \begin{pmatrix} Q_0(z) & \hat{Q}_0(z) \\ Q_2(z) & -\hat{Q}_2(z) \end{pmatrix}. \qquad (14)$$

We impose the PR conditions on $\mathbf{E}'(z)$ by imposing the PR condition on each of its building blocks $\mathbf{A}_i(z)$ and $\mathbf{E}'_1(z)$. We will restrict $\mathbf{A}_i(z)$ to be

$$\mathbf{A}_i(z) = \frac{1}{\alpha_i^2 - 1}\mathbf{C}_i\Lambda(z), \qquad (15)$$

where $\mathbf{C}_i = \begin{pmatrix} \alpha_i & 1 \\ 1 & \alpha_i \end{pmatrix}$ and $\Lambda(z) = \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix}$. We will show later that the above form for $\mathbf{A}_i(z)$ is sufficient to cover most PR LP analysis filters. The above form $\mathbf{A}_i(z)$ is an acceptable building block for a PR structure since $\det \mathbf{A}_i(z) = z^{-1}$. To avoid singularity, we constraint the value of α_i such that $\alpha_i^2 \neq 1$ in the design process. The general form of $\mathbf{E}'_1(z)$ in (14) with first order entries is:

$$\mathbf{E}'_1(z) = \begin{pmatrix} a_0 + a_1 z^{-1} & a_1 + a_0 z^{-1} \\ b_0 + b_1 z^{-1} & -b_1 - b_0 z^{-1} \end{pmatrix} \qquad (16)$$

Dividing $\mathbf{E}'_1(z)$ by a_0, computing its determinant, equating it with z^{-1} and comparing the like powers, (16) yields

$$\mathbf{E}'_1(z) = \begin{pmatrix} 1 + bz^{-1} & b + z^{-1} \\ \frac{D}{2}(1 - bz^{-1}) & \frac{D}{2}(b - z^{-1}) \end{pmatrix} \qquad (17)$$

where $D^{-1} = b^2 - 1, b^2 \neq 1$. With $\mathbf{A}_i(z)$ and $\mathbf{E}'_1(z)$ as in (15) and (17) respectively, the resulting structure of a two channel LP PR QMF analysis bank is shown in Fig. 2 where $\beta = \prod_{i=0}^{S-1} \frac{1}{(\alpha_i^2 - 1)}$ and S is the number of sections.

We have derived a lattice structure of a two channel PR QMF analysis bank which yields LP filters. Given $\mathbf{E}'_S(z)$ of order S as in (4), i.e.,

$$\mathbf{E}'_S(z) = \begin{pmatrix} Q_0^{(S)}(z) & \hat{Q}_0^{(S)}(z) \\ Q_2^{(S)}(z) & -\hat{Q}_2^{(S)}(z) \end{pmatrix}, \qquad (18)$$

can we find α_i and b for the building blocks in (15) and (17) such that $\mathbf{E}'_S(z) = \mathbf{E}'_1(z)\mathbf{A}_{S-1}(z) \ldots \mathbf{A}_0(z)$? The answer is in the affirmative under some conditions and we proceed to justify it. We synthesize $\mathbf{E}'_1(z)$ by first observing that

$$\prod_{i=0}^{S-1} \mathbf{A}_{S-1-i}(z) = (\mathbf{E}'_1(z))^{-1}\mathbf{E}'_S(z) \qquad (19)$$

$$= \begin{pmatrix} \frac{D}{2}(bz - 1) & -(bz + 1) \\ -\frac{D}{2}(z - b) & (z + b) \end{pmatrix} \begin{pmatrix} Q_0^{(S)}(z) & \hat{Q}_0^{(S)}(z) \\ Q_2^{(S)}(z) & -\hat{Q}_2^{(S)}(z) \end{pmatrix}.$$

Denoting the coefficients of the k^{th} power of $Q_i^{(S)}(z)$ by $q_{i,k}^{(S)}$, and comparing like-powers of z in (19), we obtain $\frac{D}{2} = \frac{q_{2,0}^{(S)}}{q_{0,0}^{(S)}} = -\frac{q_{2,S}^{(S)}}{q_{0,S}^{(S)}}$. This equality is consistent with the requirement that $\det \mathbf{E}'_S(z)$ be a delay (which requires that $-(q_{2,0}^{(S)}\, q_{0,S}^{(S)} + q_{0,0}^{(S)}\, q_{2,S}^{(S)}) = 0$.) Thus,

$$b^2 = 1 + \frac{q_{0,0}^{(S)}}{2q_{2,0}^{(S)}} = 1 - \frac{q_{0,S}^{(S)}}{2q_{2,S}^{(S)}}. \qquad (20)$$

Having synthesized $\mathbf{E}'_1(z)$, the product in (19) has the same form as $\mathbf{A}_i(z)$ in (14). Define $\mathbf{A}^{(m+1)}(z) = \mathbf{A}_{m+1}(z) \ldots \mathbf{A}_0(z)$. Given $\mathbf{A}^{(m+1)}(z)$, we would like to find $\mathbf{A}_{m+1}(z)$ such that $\mathbf{A}^{(m+1)}(z) = \mathbf{A}_{m+1}(z)\mathbf{A}^{(m)}(z)$ or $\mathbf{A}^{(m)}(z) = \mathbf{A}_{m+1}^{-1}(z)\ \mathbf{A}^{(m+1)}(z)$ where $\mathbf{A}^{(m+1)}(z)$ and $\mathbf{A}^{(m)}(z)$ have the same form as in (14), and $\mathbf{A}_{m+1}(z)$ has the form as in (15).

$$\mathbf{A}^{(m)}(z) = \begin{pmatrix} \alpha_{m+1} & -1 \\ -z & z\alpha_{m+1} \end{pmatrix} \begin{pmatrix} A_{00}^{(m+1)}(z) & \hat{A}_{10}^{(m+1)}(z) \\ A_{10}^{(m+1)}(z) & \hat{A}_{00}^{(m+1)}(z) \end{pmatrix}.$$

Denoting the coefficient of the k^{th} powers of $A_{ij}^{(m+1)}(z)$ by $a_{ij,k}^{(m+1)}$, and equating the coefficients of z in the above equation, we have

$$\alpha_{m+1} = \frac{a_{00,0}^{(m+1)}}{a_{10,0}^{(m+1)}} = \frac{a_{10,m+1}^{(m+1)}}{a_{00,m+1}^{(m+1)}}. \tag{21}$$

The above equality is consistent with the requirement that det $\mathbf{A}^{(m+1)}(z)$ be a delay (which requires that $a_{00,0}^{(m+1)} a_{00,m+1}^{(m+1)} - a_{10,0}^{(m+1)} a_{10,m+1}^{(m+1)} = 0$.) The condition for the above synthesis procedure to work is that $a_{00,0}^{(m+1)} \neq a_{10,0}^{(m+1)}$ or equivalently, $\alpha_{m+1}^2 \neq 1$. We have thus outlined a synthesis procedure for most pairs of PR LP analysis filters. As a design example, Fig. 3 shows the frequency response of the optimized linear phase analysis filters $H_0(z)$ and $H_1(z)$ of order 61, with bandedges at 0.4π and 0.6π respectively.

Comments: Let $H_0(z)$ and $H_1(z)$ be any two linear phase filters which have the same order. Furthermore, let $H_0(z)$ and $H_1(z)$ be symmetric and anti-symmetric respectively. In the majority of cases, we can synthesize them as in Fig. 4 where $\mathbf{C}_i = \begin{pmatrix} \alpha_i & 1 \\ 1 & \alpha_i \end{pmatrix}$ and $\mathbf{E}'_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$. The precise conditions will not be elaborated here. Denoting the two transfer functions before the $\mathbf{C}_i$ blocks in Fig. 4 by $T_i(z)$ and $U_i(z)$, it can be verified that they are mirror images, i.e., $U_i(z) = \tilde{T}_i(z)$. If the filters are PR, then every other block $\mathbf{C}_i$ degenerates into the identity matrix $\mathbf{I}$ and thus, the structure in Fig. 4 simplifies to the one in Fig. 2.

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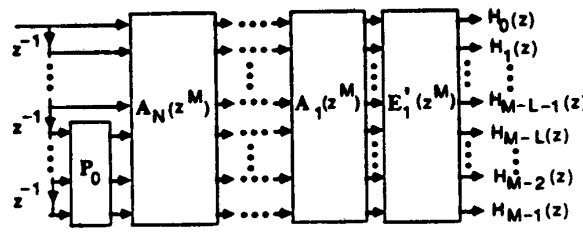

Fig. 1 Analysis bank for structure that yields LP filters.

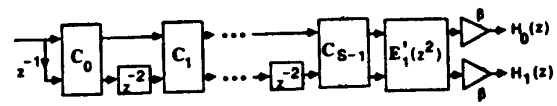

Fig. 2 Two channel LP PR QMF analysis bank.

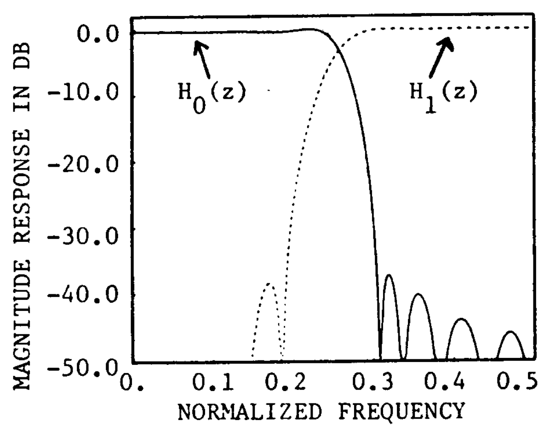

Fig. 3 Magnitude responses plot for the optimized LP PR analysis filters.

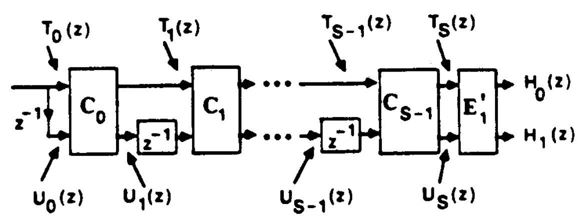

Fig. 4 General structure for pairs of LP filters.